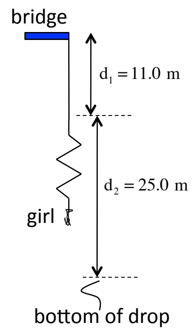
# Problem 15.19

A "crazy" girl jumps off a bridge doing a bungee jump (crazy on in the sense that she was having crazy fun). She freefalls 11.0 meters before the cord begins to elongate after which she drops and additional 25.0 meters (due to the elastic cord) before stopping (then bouncing back up).




a.) What model describes the first part of her fall?

It's free-fall under the influence of a constant gravitational force.

b.) How long does she free-fall?

You could use energy, but kinematics will work just as well. With "y=0" as defined at the bottom of the free fall:

$$y_2 = y_1 + \cancelto{0}{v_{1,y}(\Delta t)} + \frac{1}{2} a_y (\Delta t)^2$$

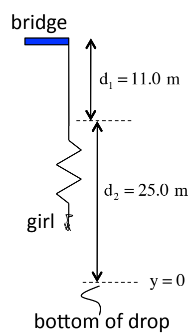
$$0 = (11.0\text{ m}) + 0 + \frac{1}{2}(-9.80\text{ m/s}^2)(\Delta t)^2$$

$$\Rightarrow \quad t = 1.50\text{ s}$$



c.) Is the system isolated?

Assuming there is no wind and we are ignoring air friction, which would under normal conditions vary with the velocity, the system is isolated.



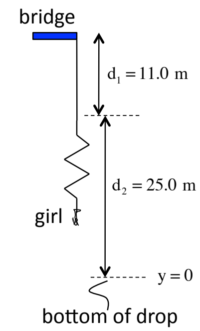

d.) Using the information from *Part c,* determine the spring constant.

Translation: The information from *Part c* suggests that you can use *conservation of energy*.

Note: The temptation might be to use *conservation of energy* just through the bungee part, but because the system is closed, you can actually start at the bridge and go to the bottom of the jump. So with y = 0 at the bottom:



*Conservation of energy* yields:




$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mg(d_1 + d_2) + 0 = 0 + \frac{1}{2}k(d_2)^2$$

$$\Rightarrow \quad k = \frac{2mg(d_1 + d_2)}{(d_2)^2}$$

$$= \frac{2(65\text{ kg})(9.80\text{ m/s}^2)((11.0\text{ m}) + (25.0\text{ m}))}{(25.0\text{ m})^2}$$

$$= 73.4\text{ N/m}$$

e.) What is the springs equilibrium point (i.e., where gravity and the spring force balance on another)?

This is a N.S.L. problem:



The balancing point, relative to where the spring began its influence 11.0 meters below the bridge, is:

$$\sum F_y:$$

$$-mg + kx = m\cancelto{0}{a}$$

$$\Rightarrow \quad (65\text{ kg})(9.80\text{ m/s}^2) = (73.4\text{ N/m})x$$

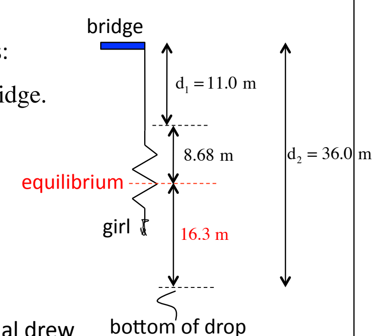
$$\Rightarrow \quad x = 8.68\text{ m}$$

That means that relative to the bridge, the point is:

$$(8.68\text{ m}) + (11.0\text{ m}) = (19.7\text{ m})\text{ below the bridge.}$$

The low point on the oscillation is 36.0 meters below the bridge, so with the equilibrium at 19.7 meters below the the bridge, the oscillation must have an amplitude of

$$(36.0\text{ m}) - (19.7\text{ m}) = 16.3\text{ meters.}$$

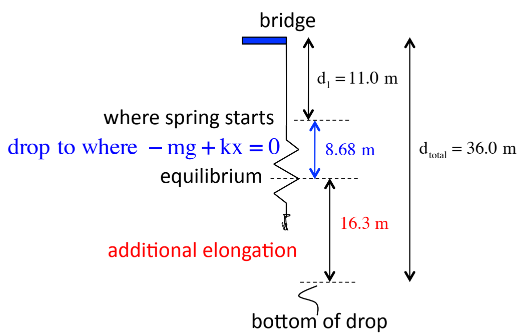



(Note that this is the way the text's Solution Manual drew conclusions.)

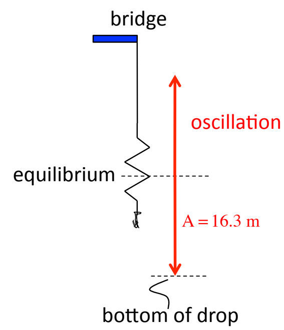
This is definitely obscure and assumes some stuff that isn't probably obvious. Follow along:

If there hadn't been any initial velocity and the girl was slowly lowered to her equilibrium position, *then* additionally lowered to the 36.0 meter point and released to oscillate, the situation would have looked as pictured on the left and, summarily, on the right.

SET-UP

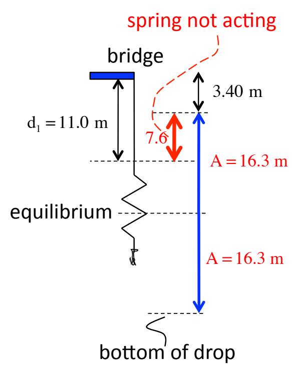



FINAL OSCILLATION






When you think about an oscillation produced by a spring, you assume the spring is engaged over the entire motion. In this case, apparently, the amplitude of the motion is A = 16.3 meters. Twice that (to find the maximum height achieved as the body moved "A" units above equilibrium) is 32.6 meters, which is only 3.4 meters below the bridge. But the spring doesn't engage until 11.0 meters below the bridge, so evidently the only force acting over the top 11.0 - 3.4 = 7.6 meters is gravity. VERY FUNKY!




f.) What is the angular frequency for the oscillation?

$$\omega = \sqrt{\frac{k}{m}}$$

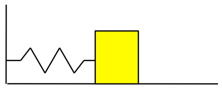
$$= \sqrt{\frac{(73.4 \text{ N/m})}{(65 \text{ kg})}}$$

$$= 1.06 \text{ rad/s}$$



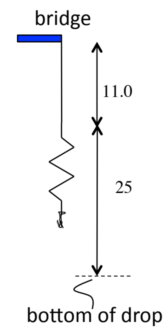
g.) What is the time required for the cord to stretch 25.0 meters?

Warning: To understand this problem, you are going to have to actively think through the problem on your own. I will provide you with commentary, but if you don't do some active thinking yourself, it won't help much.



Background: Think for a moment about a standard, table top spring/mass problem. When the spring is at its equilibrium position, all forces in the system along the line of motion have been nulled out and, when released, the mass will do just sit there. It isn't until the mass is displaced some distance "x" from the equilibrium position that a force *provided by the spring* is exerted on the mass. For the analysis to make sense, then, everything is measured relative to that equilibrium position.

Now consider our problem. We know how long it take the girl to free fall the first 11.0 meters (we calculated that earlier). What we need here is the time it takes to go that last 25 meters.

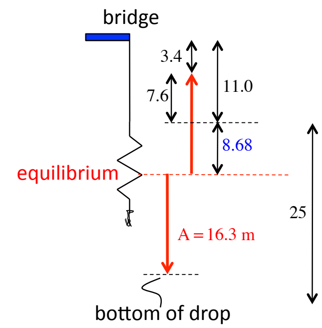





You might think that that would be a trivial task. It isn't because technically, both gravity and the spring are acting over that last 14 meters. So what to do?

The plan is to do the following. If we let the girl down slowly to the point where gravity and the spring force null out, which is to say, if we let her lower to the the spring/gravity equilibrium point, any subsequent displacement of the girl will produce a force that is due solely to the spring (this is just like the horizontal spring situation where the equilibrium position signified the point where the net force is nulled out). Any additional displacement of the girl from that equilibrium position will produce an up and down oscillation which will be simple harmonic in nature.

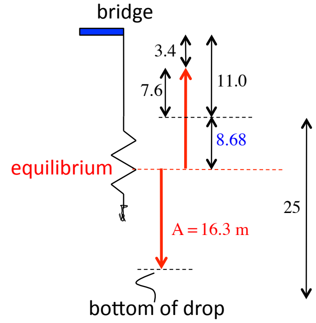
OK? Now look at the distances in the sketch. You can see the net 25 meter distance displayed. You can see the upper 11.0 meters drop the time of which we know. You can see where the spring/ mass system's equilibrium position is, and in red what the motion's amplitude above and below equilibrium will be ($\pm$16.3 meters).



The girl will drop the first 11.0 meters (again, we know how long that will take). She will then drop an additional 8.68 meters *under the influence of the spring only* (as we are measuring that 8.68 meters from the system's equilibrium point, the spring and gravity have already nulled one another out and it's only the spring doing the extra stuff) to the equilibrium point, then continue on for another 16.3 meters beyond that equilibrium point (also effectively under the sole influence of the spring).

Sooo, if we could derive an expression for the girl's position as a function of time, we should be able to determine the time it takes for her to go from 8.68 meters above equilibrium to 16.3 meters below equilibrium. In other words (and this is getting exciting), to do the problem we have to derive an expression for the position of the girl as a function of time, complete with phase shift.

I have presented the oscillation (again, in red) to the right. I have additionally included all the distances involved in the motion.

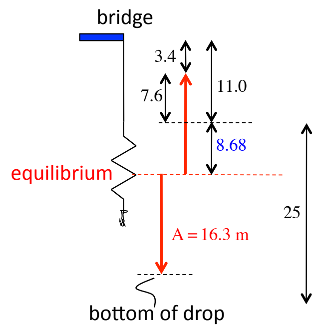





We will start with your standard sine wave (i.e., one in which the displacement is y = 0 at t = 0, with the subsequent motion moving into the "+y" region). I am going to assume that moving downward away from equilibrium is going to take me into the *+y region* (in other words, *down* is positive), and I am going to slide the sine wave's axis until t = 0 corresponds to a position -8.68 meters from equilibrium (this will be *above* equilibrium at the point where the initial 11.0 meters free fall terminates). I'm going to use the math to determine the phase shift for the motion, write out a "y(t)" expression for the motion, then use that to determine how long it takes to go from -8.68 meters at "t = 0" to +16.3 meters at "t."

Although this will answer this problem, for an added flourish I will add that time to the time it took for the girl to go the first 11.0 meters and, God willing, my answer and the text's Solution Manual answer to *Part h* will gibe . . . cause if it don't, I'm gonna kill myself.

Let's see how this works out.






Here's the math to determine the phase shift. We know that:

$$\begin{aligned} y &= A\sin(\omega t+\phi) \\ &= (16.3 \text{ m})\sin((1.06 \text{ rad/s})t+\phi) \end{aligned}$$

So at *t = 0* when *y = -8.68 meters*, we can write:

$$(-8.68 \text{ m}) = (16.3 \text{ m})\sin((1.06 \cancelto{0}{\text{rad/s}})(0)+\phi)$$

$$\Rightarrow \quad (-8.68 \text{ m}) = (16.3 \text{ m})\sin(\phi)$$

$$\Rightarrow \quad \sin(\phi) = \frac{-8.68}{16.3}$$

$$\Rightarrow \quad \phi = -.562 \text{ radians}$$

Apparently, our function is:

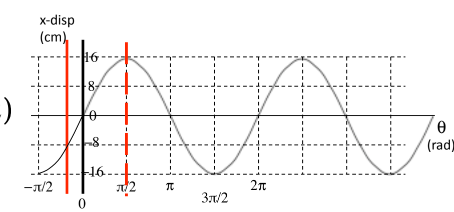
$$y(t) = (16.3 \text{ m})\sin((1.06 \text{ rad/s})t-(.562 \text{ rad})), \text{ or in bare bones:}$$

$$y = 16.3\sin(1.06t-.562)$$



Minor note: The solid red line is where the axis would have to be put to effect this situation, and the dotted red line identifies where we are going 16.3 meters below equilibrium:

$$y = 16.3\sin(1.06t-.562)$$




The time it takes to go from -8.68 meters (which is the girl's position at t=0) to 16.3 meters (for the full 25.0 meters drop) should be:

$$\begin{aligned} y_{16m} = 16.3 &= 16.3\sin(1.06t_{16m}-.562) \\ \Rightarrow \quad 1 &= \sin(1.06t_{16m}-.562) \\ \Rightarrow \quad 1.06t_{16m}-.562 &= \sin^{-1}(1.00) \\ \Rightarrow \quad 1.06t_{16m}-.562 &= 1.57 \\ \Rightarrow \quad 1.06t_{16m} &= 2.13 \\ \Rightarrow \quad t_{16m} &= 2.01 \text{ s} \end{aligned}$$

h.) What is the total time of fall?

$$\begin{aligned} t_{tot} &= t_{11} + t_{16m} \\ &= (1.50 \text{ s}) + (2.01 \text{ s}) \\ &= 3.51 \text{ s} \end{aligned}$$

This is the book's solution, and you folks are lamentably going to have to find another way to knock me off . . .